The Geometry of Interaction of Differential Interaction Nets

choco

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Institut de Mathématiques de Luminy

Logic in Computer Science 08

Outline

We study

- differential interaction nets (din) : extension of linear logic [Ehrhard and Regnier, 2005], presented as formal sums of graph-like structures and rewriting, encoding resource λ-calculus and a finitary π-calculus
- geometry of interaction (GoI) : a special kind of semantics accounting for reduction, akin to game semantics, defined on fragments of linear logic [Girard, 1989],[Girard, 1995]

We extend the path based version of GoI [Danos and Regnier, 1995], i.e. we

- define a proper notion of paths
- define a proper equational theory encoding reduction in a local and asynchronous way
- prove that the theory is coherent by giving a realisation
- prove that our encoding of path reduction is sound

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Linear Logic can be seen as an explicit substitution system for λ-calculus

data is split between

offers : *arguments of application*
provided as a factory producing exact copies of the same object

demands : *occurrences of variables*
organized as a tree of demands

Mass production issues: *non personalized offer, not fault-tolerant, . . .*
Can we replace it with craftsmanship?

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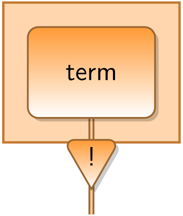

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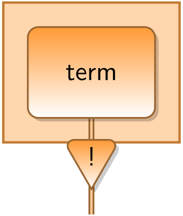

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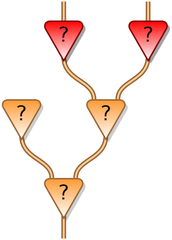

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Linear Logic from a calculus point of view

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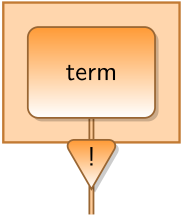

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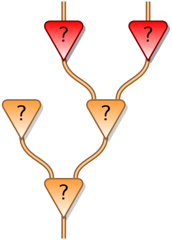

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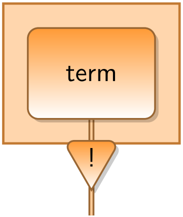

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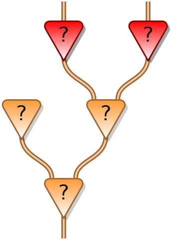

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Differential Linear Logic can be seen as an explicit substitution system for **resource** λ-calculus

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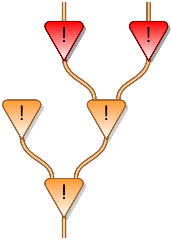

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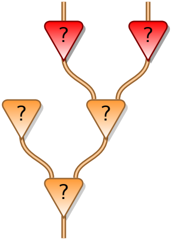

Differential Interaction Nets

- the natural presentation of differential linear logic akin to proof-net of linear logic
- a special kind of interaction nets using the cells

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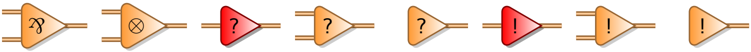

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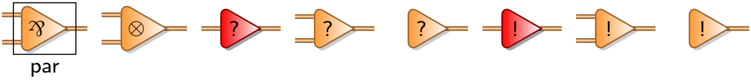

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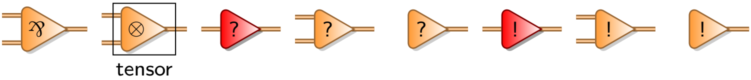

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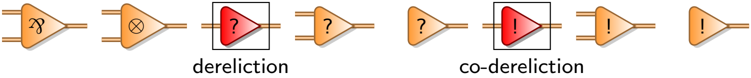

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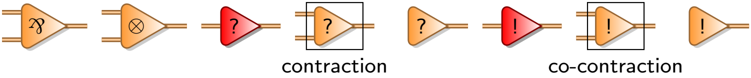

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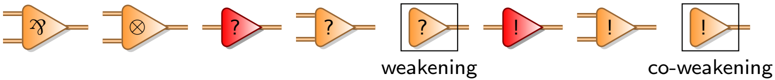

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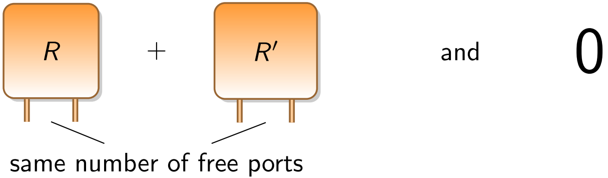

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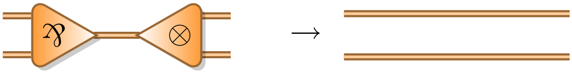
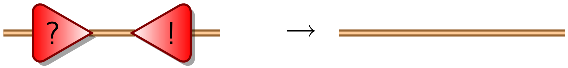
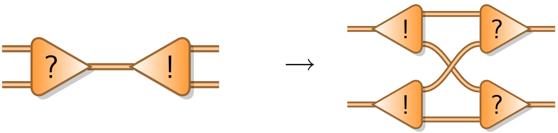


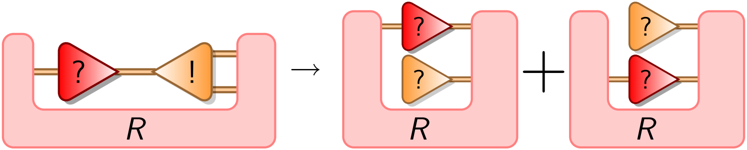
- ?/! rules: we only present half of them, others obtained by duality
- dereliction/co-dereliction : *offer meets demand*



- contraction/co-contraction : *independent routing*



dereliction/co-contraction : *one demand for a binary offer*



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Goal of a GoI

Can we replace this global reduction with a lot of local sum propagations?

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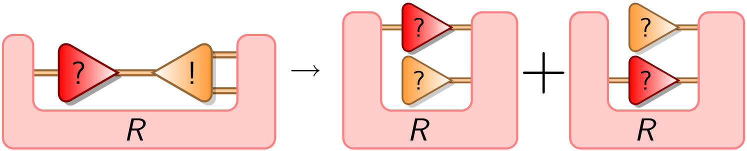

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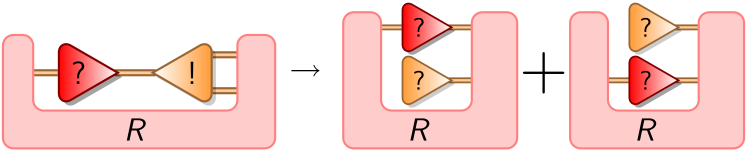

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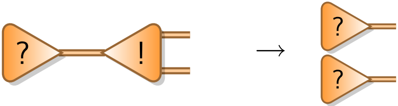
weakening/co-contraction : *routing a void demand*



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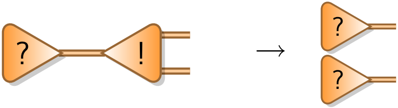

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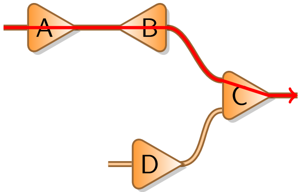
- standard notion of path coming from the graph-like structure



- a reduction $R \longrightarrow R'$ extends to a reduction from $\mathcal{P}(R) \longrightarrow \mathcal{P}(R')^\star$
- a path can be destroyed

- *persistent path*: a path not destroyed by any chain of reduction
- *GoI goal*: find a structure S and a morphism $w : \mathcal{P}(R) \longrightarrow S$ such that

$$\varphi \text{ persistent} \iff w(\varphi) \neq 0$$

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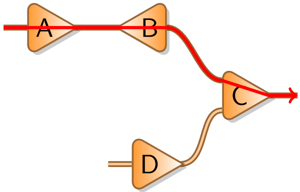

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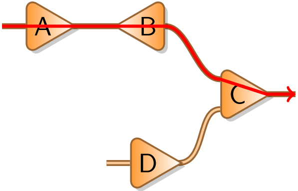

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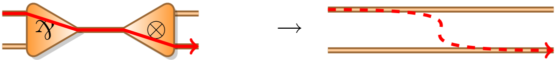

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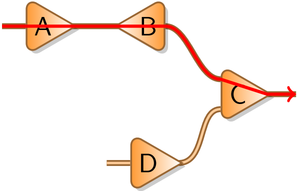

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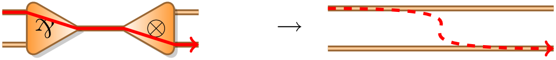

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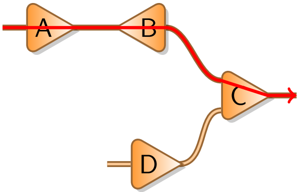

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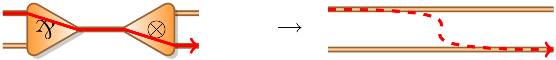

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- What is a path in $R + R'$? How can we distinguish a path in one the Rs of $R + R$?
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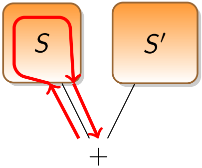

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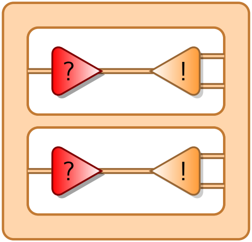

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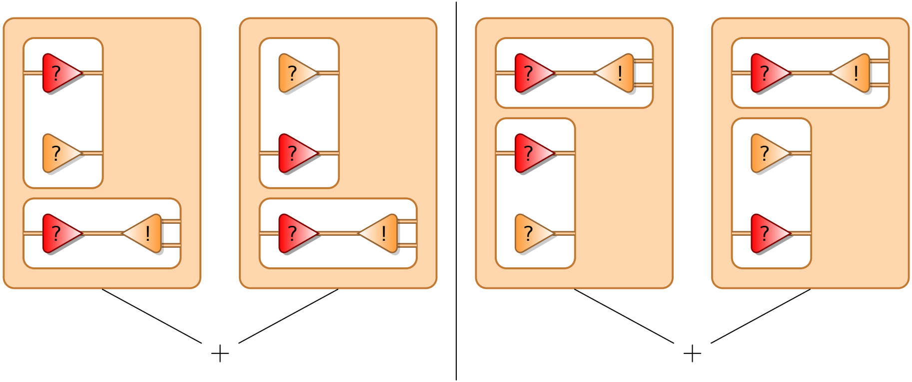

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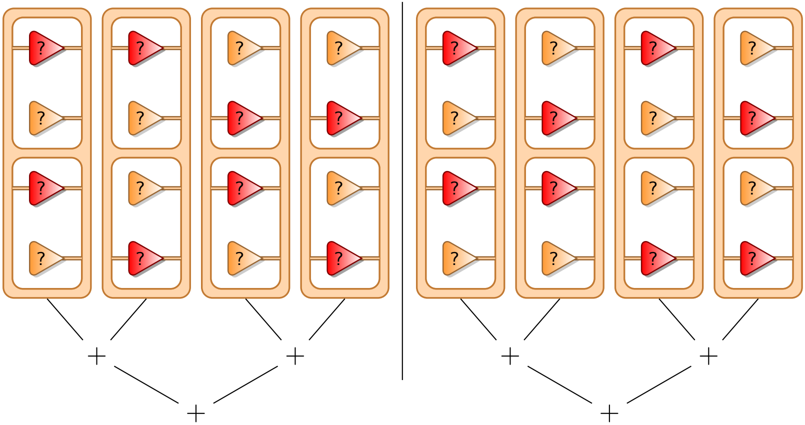

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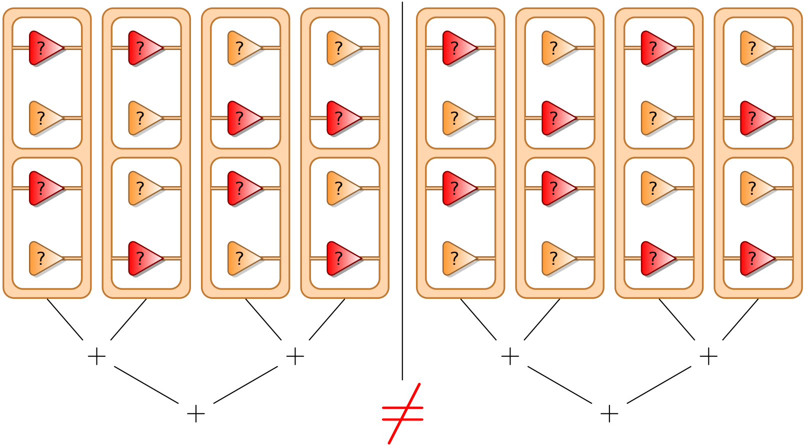

Names

We add names to track down the context of sum production

- we give a unique name to each (co)dereliction in a simple net, and we label the nodes of the tree
- we replace the sum producing rules by

- we add middle-four interchange law between $+_\alpha$ and $+_\beta$ for $\alpha \neq \beta$:

$$(R_1 +_\alpha R_2) +_\beta (R_3 +_\alpha R_4) \equiv (R_1 +_\beta R_3) +_\alpha (R_2 +_\beta R_4)$$

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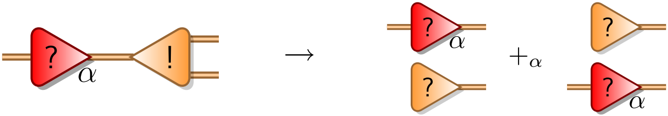

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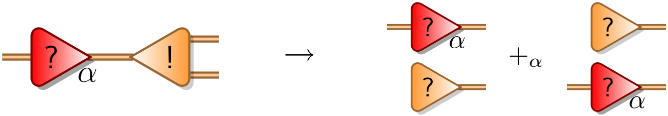
$$(R_1 +_\alpha R_2) +_\beta (R_3 +_\alpha R_4) \equiv (R_1 +_\beta R_3) +_\alpha (R_2 +_\beta R_4)$$

Names

We add names to track down the context of sum production

- we give a unique name to each (co)dereliction in a simple net, and we label the nodes of the tree
- we replace the sum producing rules by



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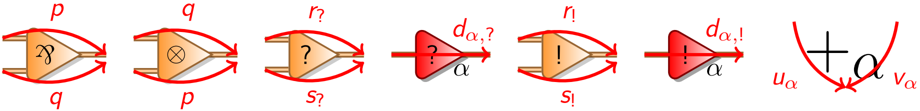
we construct an inverse monoid with zero $\partial L^\star$ and a weighting of path w with

$$w(\varphi\varphi') = w(\varphi')w(\varphi) \quad w(\rightarrow) = w(\leftarrow)^\star$$

$\partial L^\star$

we weight cell traversals with generators:



and relations :

- (p, q) and (u_α, v_α) like MLL : $p^\star p = 1, q^\star p = 0, \ldots$
- $r_!, r_?, s_!, s_?$ have bigebras relations : $r_!^\star s_? = s_? r_!^\star, \ldots$
- $d^\star_{\alpha,!} d_{\beta,?}$
- $u_\alpha, v_\alpha, e_\alpha$ commutes with everything non-α
- $d^\star_{\alpha,t} s_{t'} = u_\alpha d^\star_{\alpha,t} u^\star_\alpha$, $d^\star_{\alpha,t} r_{t'} = v_\alpha d^\star_{\alpha,t} v^\star_\alpha$ where $t \neq t'$, $\alpha \neq \beta$

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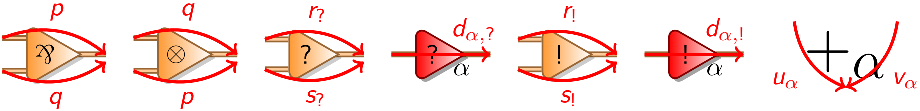

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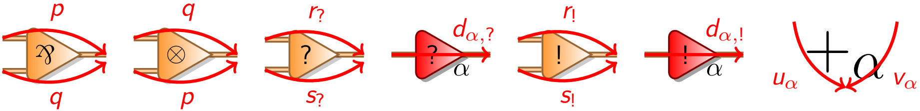

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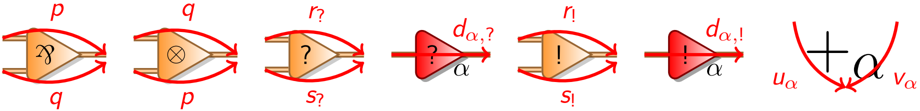

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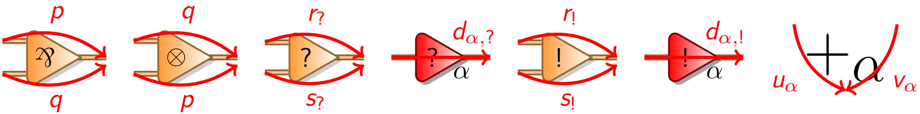

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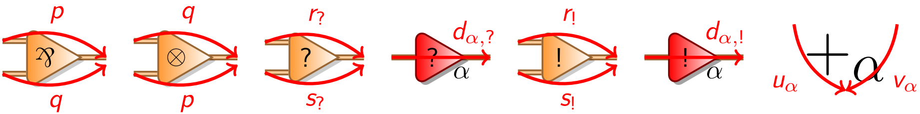

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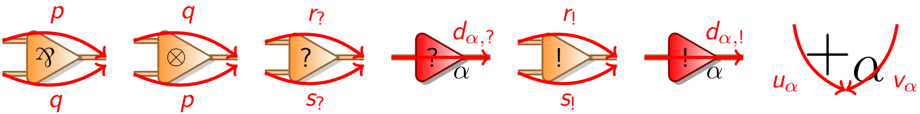

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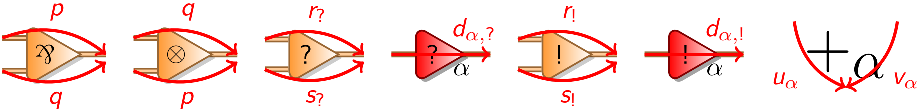

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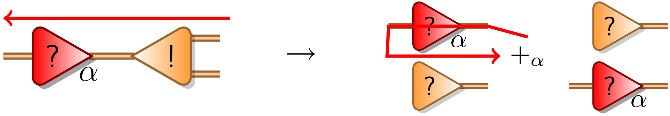


Fact

$\partial L^\star$ is non-trivial: $0 \neq 1$

Proof.

by constructing a non-trivial realization as a operations on concrete objects: *tokens* made of stacks,. . . □

Theorem

φ is weakly-persistent $\iff$ $w(\varphi) \neq 0$

normalizing factors:

$$n_\varphi(\varphi') = \prod_{\alpha \in \varphi, \alpha \notin \varphi'} e_\alpha$$

we have equality along weak-reduction up to normalizing factors:

Lemma (fundamental lemma)

$R \to R'$ weakly, $\varphi \in \mathcal{P}(R)$ deformed by reduction

- *either $\varphi \to \varphi'$ and $w(\varphi) = n_\varphi(\varphi')w(\varphi')$*
- *or φ destroyed and $w(\varphi) = 0$*

Weights structure

We recover the Danos-Regnier $ab^\star$ theorem of GoI:

Theorem ($\alpha ab^\star \alpha^\star$)

$w(\varphi) = 0$ *or* $\exists \alpha \in \partial {L^\star}^+_a, a, b \in \partial {L^\star}^+_{me}$ *and* $w(\varphi) = \alpha ab^\star \alpha^\star$

where we distinguish two monoids:

- $\partial {L^\star}^+_{me}$ generated by $\{p, q, r, s, d\}$
- $\partial {L^\star}^+_a$ generated by $\{u, v, e\}$

Normalized Execution

We consider formal sums of weights and set

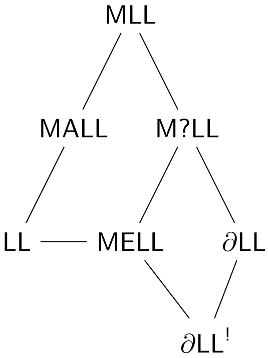
$$\mathrm{NEX}_{R_0}(R) = \sum_{\varphi \in R} n_{R_0}(\varphi) w(\varphi)$$

Theorem

NEX_{R_0} is an invariant of weak reductions starting from R_0

Expressive power of the equational theory

$\partial L^\star$ is rich enough to encode more than ∂LL



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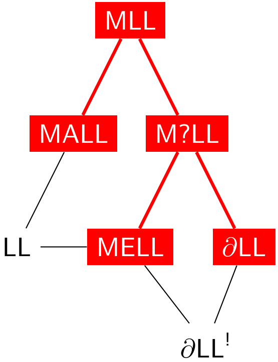

for MELL we have to extend a bit $\partial L^\star$, but the extension is also non-trivial

Expressive power of the equational theory

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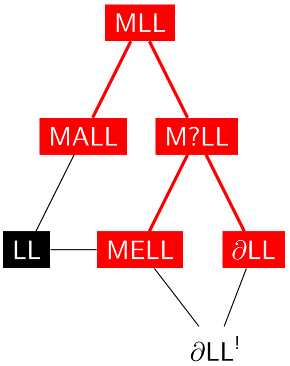

out of reach because of the interactions between exponentials and additives

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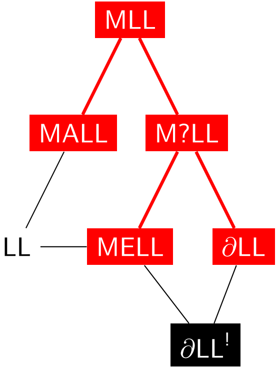

out of reach because of the strong use of commutativity of links [Tranquilli, 2007]

Future works

- Non-deterministic computation
  - differential interaction nets can encode a finitary π-calculus, thus giving a notion of paths and GoI for this calculus
  - the notion of sub-tree can be made compatible with the sum interchange law: *slices*
  - the weight of slices is a lattice, thus, we can express properties of non-determinism as computations of lower and upper bounds
- sharing graphs and readback process for dins
- AJM-style game semantics extracted from the GoI
- try to extend the GoI to $\partial \mathrm{LL}^!$, maybe by means of *variants* to cope with commutativity issues

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Questions

beamericonarticle Danos, V. and Regnier, L. (1995).
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